CBSE
Class XII Mathematics
Board Paper
2023

Time: 3 Hours **Total Marks: 80**

Read the following instructions very carefully and strictly follow them:

(i) This question paper contains **38** questions. **All** questions are **compulsory**.
(ii) This question paper is divided into **five** sections – **A, B, C, D** and **E**.
(iii) In **Section A**, Questions no. **1** to **18** are multiple choice questions (MCQs) and questions number **19** and **20** are Assertion-Reason based questions of **1** mark each.
(iv) In **Section B**, Questions no. **21** to **25** are very short answer (VSA) type questions, carrying **2** marks each.
(v) In **Section C**, Questions no. **26** to **31** are short answer (SA) type questions, carrying **3** marks each.
(vi) In **Section D**, Questions no. **32** to **35** are long answer (LA) type questions, carrying **5** marks each.
(vii) In **Section E**, Questions no. **36** to **38** are case study based questions carrying **4** marks each.
(viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in section D and 2 questions in Section E
(ix) Use of calculators is **not** allowed.

SECTION A

This section comprises multiple-choice questions (MCQs) of 1 mark each.

Question 1

If $A = \begin{bmatrix} 1 & 4 & x \\ z & 2 & y \\ -3 & -1 & 3 \end{bmatrix}$ is a symmetric matrix, then the value of x + y + z is: [1]

(a) 1
(b) 6
(c) 8
(d) 0

Question 2

If $A \cdot (\text{adj } A) = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$, then the value of |A| + |adj A| is equal to: [1]

(a) 12
(b) 9
(c) 3
(d) 27

Question 3

A and B are skew symmetric matrices of same order. AB is symmetric, if: [1]

(a) AB = 0

(b) AB = -BA

(c) AB = BA

(d) BA = 0

Question 4

For what value of $x \epsilon \left[0, \frac{\pi}{2}\right]$, is A + A' = $\sqrt{3}$ I, where $A = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$? [1]

(a) $\frac{\pi}{3}$

(b) $\frac{\pi}{6}$

(c) 0

(d) $\frac{\pi}{2}$

Question 5

Let A be the area of a triangle having vertices (x_1, y_1), (x_2, y_2) and (x_3, y_3) which of the following is correct? [1]

(a) $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \pm A$

(b) $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \pm 2A$

(c) $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \pm \frac{A}{2}$

(d) $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}^2 = A^2$

Question 6

$\int 2^{x+2}\,dx$ is equal to: [1]

(a) $2^{x+2} + c$

(b) $2^{x+2}\log 2 + c$

(c) $\frac{2^{x+2}}{\log 2} + c$

(d) $2 - \frac{2^{x}}{\log 2} + c$

Question 7

$\int \frac{2\cos 2x - 1}{1 + 2\sin x}\,dx$ is equal to : [1]

(a) $x - 2\cos x + C$

(b) $x + 2\cos x + C$

(c) $-x - 2\cos x + C$

(d) $-x + 2\cos x + C$

Question 8

The solution of the differential equation $\frac{dx}{x} + \frac{dy}{y} = 0$ is: [1]

(a) $\frac{1}{x} + \frac{1}{y} = c$

(b) $\log x - \log y = c$

(c) $xy = c$

(d) $x + y = c$

Question 9

What is the product of the order and degree of the differential equation

$\frac{d^2y}{dx^2}\sin y + \left(\frac{dy}{dx}\right)^3 \cos y - \sqrt{y}$? [1]

(a) 3

(b) 2

(c) 6

(d) Not defined

Question 10

If a vector makes an angle of $\frac{\pi}{4}$ with the positive directions of both x-axis and y axis, then the angle which it makes with positive z-axis is: [1]

(a) $\frac{\pi}{4}$

(b) $\frac{3\pi}{4}$

(c) $\frac{\pi}{2}$

(d) 0

Question 11

a and b are two non-zero vectors such that the projection of a on b is 0. The angle between a and b is: [1]

(a) $\frac{\pi}{2}$

(b) π

(c) $\frac{\pi}{4}$

(d) 0

Question 12

In ΔABC, $AB = i + j + 2k$ and $AC = 3i - j + 4k$. If D is mid-point of BC, then vector AD is equal to: [1]

(a) $4i + 6k$

(b) $2i - 2j + 2k$

(c) $i - j + k$

(d) $2i + 3k$

Question 13

The value of λ for which the angle between the lines [1]

$r = i + j + k + p\left(2i + j + 2k\right)$ and

$r = \left(1 + q\right)k + \left(1 + q\lambda\right)j + \left(1 + q\right)k$ is $\frac{\pi}{2}$ is:

(a) −4

(b) 4

(c) 2

(d) −2

Question 14

If $P(A \cap B) = \frac{1}{8}$ and $P(\overline{A}) = \frac{3}{4}$, then $P\left(\frac{B}{A}\right)$ is equal to: [1]

(a) $\frac{1}{2}$

(b) $\frac{1}{8}$

(c) $\frac{1}{6}$

(d) $\frac{2}{3}$

Question 15

The value of k for which function $f(x) = \begin{cases} x^2, & x \geq 0 \\ kx & x < 0 \end{cases}$ is differentiable at x = 0 is: [1]

(a) 1
(b) 2
(c) any real number
(d) 0

Question 16

If $y = \frac{\cos x - \sin x}{\cos x + \sin x}$, then $\frac{dy}{dx}$ is: [1]

(a) $-\sec^2\left(\frac{\pi}{4} - x\right)$

(b) $\sec^2\left(\frac{\pi}{4} - x\right)$

(c) $\log\left|\sec\left(\frac{\pi}{4} - x\right)\right|$

(d) $-\log\left|\sec\left(\frac{\pi}{4} - x\right)\right|$

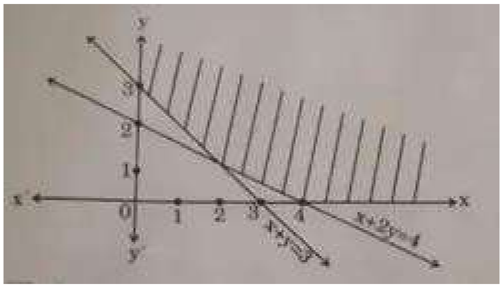
Question 17

The number of feasible solutions of the linear programming problem given as
Maximise Z = 15x + 30y subject to constraints:
$3x + y \leq 12, x + 2y \leq 10, x \geq 0, y \geq 0$ is [1]

(a) 1
(b) 2
(c) 3
(d) infinite

Question 18

The feasible region of a linear programming problem is shown in the figure below:



Which of the following are the possible constraints? [1]

(a) $x + 2y \geq 4, x + y \leq 3, x \geq 0, y \geq 0$

(b) $x + 2y \leq 4, x + y \leq 3, x \geq 0, y \geq 0$

(c) $x + 2y \geq 4, x + y \geq 3, x \geq 0, y \geq 0$

(d) $x + 2y \geq 4, x + y \geq 3, x \geq 0, y \geq 0$

Question under 19 and 20 are Assertion and Reasoning based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the (a), (b), (c) and (d) as given below.

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
(b) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
(c) Assertion (A) is true and Reason (R) is false.
(d) Assertion (A) is false and Reason (R) is true.

Question 19

Assertion (A): Range of $[\sin^{-1}x + 2\cos^{-1}x]$ is $[0, \pi]$.

Reason (R): Principal value branch of $\sin^{-1}x$ has range $[-\frac{\pi}{2}, \frac{\pi}{2}]$.

Question 20

Assertion (A): A line through the points (4, 7, 8) and (2, 3, 4) is parallel to a line through the points (–1, –2, 1) and (1, 2, 5).

Reason (R): Lines $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu\vec{b}_2$ are parallel if $\vec{b}_1 \cdot \vec{b}_2 = 0$.

Section B

This section comparison very short answer (VSA) type questions of 2 marks each.

Question 21

If $\vec{r} = 3\hat{i} - 2\hat{j} + 6\hat{k}$, find the value of $(\vec{r} \times \hat{j}) \cdot (\vec{r} \times \hat{k}) - 12$.

Question 22

If the angle between the lines $\frac{x-5}{\alpha} = \frac{y+2}{-5} = \frac{z+\frac{24}{5}}{\beta}$ and $\frac{x}{1} = \frac{y}{0} = \frac{z}{1}$ is $\frac{\pi}{4}$. Find the relation between α and β.

Question 23

If f(x) = a(tan x – cot x), where a > 0, then find whether f(x) is increasing or decreasing function in its domain.

Question 24

(a) Evaluate : $3\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) + 2\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) + \cos^{-1}(0)$

OR

(b) Draw the graph of f(x) = $\sin^{-1}x$, $x = \left[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]$, also write range of f(x).

Question 25

(a) If y = $x^{\frac{1}{x}}$, then find $\frac{dy}{dx}$ as x = 1

OR

(b) If x = a sin 2x, y = a (cos 2t + log tan t), then find $\frac{dy}{dx}$

SECTION C

This section comprises short answer (SA) type questions of 3 marks each.

Question 26

(a) Find the general solution of the differential equation:

$$\frac{d}{dx}(xy^2) = 2y(1 + x^2)$$

OR

(b) Solve the following differential equation:

$$xe^{\frac{y}{x}} - y + x\frac{dy}{dx} = 0$$

Question 27

Evaluate:

$$\int_1^3 \frac{\sqrt{4-x}}{\sqrt{x}+\sqrt{4-x}}\,dx$$

Question 28

Evaluate:

$$\int_1^e \frac{1}{\sqrt{4x^2-(x\log x)^2}}\,dx$$

Question 29

(a) Find :

$$\int \frac{\cos x}{\sin 3x}\,dx$$

OR

(b) Find :

$$\int x^2 \log\left(x^2+1\right) dx$$

Question 30

Determine graphically the minimum value of the following objective function:

Z = 500x + 400y

Subject to constraints

x + y ≤ 200,

x ≥ 20,

y ≥ 4x,

y ≥ 0

Question 31

(a) A pair of dice is thrown simultaneously. If X denote the absolute difference of numbers obtained on the pair of dice, then find the probability distribution of X.

OR

(b) There are two coins. One of them is a biased coin such that P(head) : P(tail) is 1 : 3 and the other coin is a fair coin. A coin is selected at random and tossed once. If the coin showed head, then find the probability that it is a biased coin.

SECTION D

This section comprises long answer (LA) type questions of 5 marks each.

Question 32

Show that a function f : R → R defined as $f(x) = \frac{6x-3}{4}$ is both one-one and onto.

Question 33

The area of the region bounded by the line y = mx (m > 0), the curve $x^2 + y^2 = 4$ and the x-axis in the first quadrant is $\frac{\pi}{2}$ units. Using integration, find the value of m.

Question 34

(a) If $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$, then show that $A^3 - 6A^2 + 7A + 2I = O$

OR

(b) If $A = \begin{bmatrix} 3 & 2 \\ 5 & -7 \end{bmatrix}$, then find A^{-1} and use it to solve the following system of equations :

3x + 5y = 11, 2x – 7y = –3

Question 35

(a) Find the values of b so that the line $\frac{x-1}{2} = \frac{y-b}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = z$ are intersecting lines also, find the point of intersection of these given lines.

OR

(b) Find the equations of all the sides of the parallelogram ABCD whose vertices are A(4, 7, 8), B(2, 3, 4,), C(–1, –2, 1) and D(1, 2, 5). Also, find the coordinates of the foot of the perpendicular from A to CD.

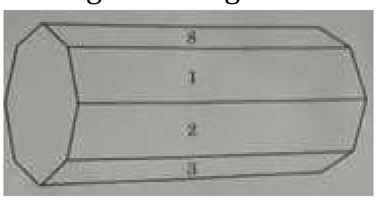
SECTION E

This section comprises 3 case study based questions of 4 marks each

Case Study - I

Question 36

An octagonal prism is a three dimensional polyhedron bounded by two octagonal bases and eight rectangular side faces. It has 224 edges and 16 vertices.



The prism is rolled along the rectangular faces and number on the bottom face (touching the ground) is noted. Let X denote the number obtained on the bottom face and the following table give the probability distribution of X.

X :	1	2	3	4	5	6	7	8
P(X):	p	$2p$	$2p$	p	$2p$	p^2	$2p^2$	$7p^2 + p$

Based on the above information, answer the following question:

i. Find the value of p

ii. Find $P(X > 6)$

iii. (a) Find $P(X = 3m)$, where m is a natural number

OR

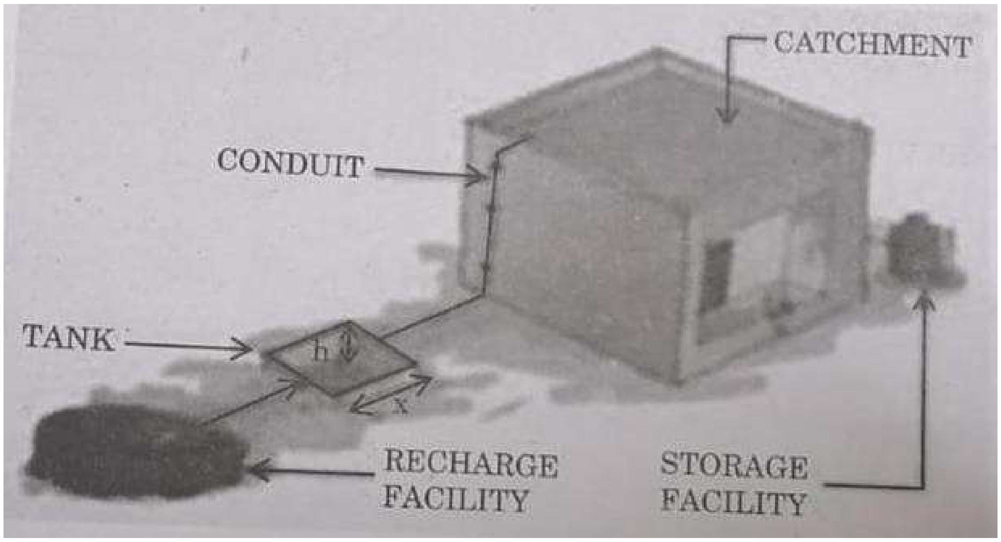
iii. (b) Find the mean E(X)

Case Study - 2

Question 37

In order to set up a rain water harvesting system, a tank to collect rain water is to be dug. The tank should save a square base and a capacity of 250 m^3. The cost of land is Rs. 5, 000 per square metre and cost of digging increases with depth and for the whole tank, it is Rs. 40,000 h^2, where h is the depth of the tank in metres, x is the side of the square base of the tank in metres.

ELEMENTS OF A TYPICAL RAIN WATER HARVESTING SYSTEM



Based on the above information, answer the following questions:

i. Find the total cost C of digging the tank in terms of x.

ii. Find $\frac{dC}{dx}$.

iii. (a) Find the value of x for which cost C is minimum.

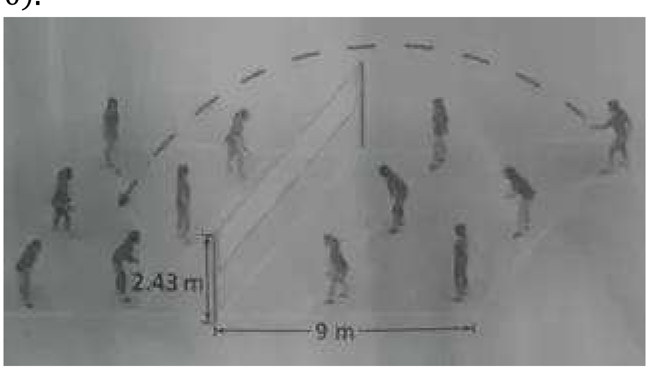
OR

iii. (b) Check whether the cost functions C(x) expressed in terms of x is increasing or not, where x > 0.

Case Study - 3

Question 38

A volleyball player serves the ball which takes a parabolic path given by the equation $h(t) = -\frac{7}{2}t^2 + \frac{13}{2}t + 1$, *where h(t) is the height of ball at any time t (in seconds)*, $(t \geq 0)$.



Based on the above information, answer the following questions:

(i) Is h(t) a continuous function? Justify.

(ii) Find the time at which the height of the hall is maximum.