**CBSE Board**
**Class XII Mathematics**
**Board Paper – 2019**
**All India Set – 1**

**Time: 3 hrs** **Total Marks: 100**

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**General Instructions:**

i. All questions are compulsory.
ii. The question paper consists of **29** questions divided into four sections: A, B, C and D. Section A comprises of **4** questions of **one mark** each, Section B comprises of **8** questions of **two marks** each, Section C comprises of **11** questions of **four marks** each and Section D comprises of **6** questions of **six marks** each.
iii. All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
iv. There is no overall choice. However, internal choice has been provided in 1 question of Section A, 3 questions of Section B, 3 questions of Section C and 3 questions of Section D.
You have to attempt only one of the alternatives in all such questions.
v. Use of calculators is **not** permitted. You may ask for logarithmic tables, if required.

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**SECTION A**

Questions number 1 to 4 carry 1 mark each.

**1.** If A is a square matrix of order 3 with |A| = 4, then write the value of |-2A|.

**2.** If $y = \sin^{-1}x + \cos^{-1}x$, find $\frac{dy}{dx}$.

**3.** Write the order and the degree of the differential equation

$$\left(\frac{d^4y}{dx^4}\right)^2 = \left[x + \left(\frac{dy}{dx}\right)^2\right]^3.$$

**4.** If a line has the direction ratios -18, 12, -4, then what are its direction cosines?

**OR**

Find the cartesian equation of the line which passes through the point (-2, 4, -5) and is parallel to the line $\frac{x+3}{3} = \frac{4-y}{5} = \frac{z+8}{6}$.

## SECTION B

Questions number 5 to 12 carry 2 marks each.

**5.** If * is defined on the set R of all real numbers by * : a * b = $\sqrt{a^2 + b^2}$, find the identity element, if it exists in R with respect to *.

**6.** If A = $\begin{bmatrix} 0 & 2 \\ 3 & -4 \end{bmatrix}$ and kA = $\begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix}$, then find the values of k, a and b.

**7.** Find:

$$\int \frac{\sin x - \cos x}{\sqrt{1 + \sin 2x}}\, dx,\ 0 < x < \pi/2$$

**8.** Find:

$$\int \frac{\sin(x - a)}{\sin(x + a)}\, dx$$

**OR**

Find:

$$\int (\log x)^2\, dx$$

**9.** Form the differential equation representing the family of curves $y^2 = m(a^2 - x^2)$ by eliminating the arbitrary constants 'm' and 'a'.

**10.** Find a unit vector perpendicular to both the vectors a and b, where a = i - 7j + 7k and b = 3i - 2j + 2k.

**OR**

Show that the vectors i - 2j + 3k, -2i + 3j - 4k and i - 3j + 5k are coplanar.

**11.** Mother, father and son line up at random for a family photo. If A and B are two events given by A = Son on one end, B = Father in the middle, find P(B/A).

**12.** Let X be a random variable which assumes values $x_1$, $x_2$, $x_3$, $x_4$ such that $2P(X = x_1) = 3P(X = x_2) = P(X = x_3) = 5P(X = x_4)$.
Find the probability distribution of X.

**OR**

A coin is tossed 5 times, Find the probability of getting (i) at least 4 heads, and (ii) at most 4 heads.

## SECTION C

Questions number 13 to 23 carry 4 marks each.

**13.** Show that the relation R on the set Z of all integers, given by R = [(a,b) : 2 divides (a - b)] is an equivalence relation.

**OR**

If $f(x) = \frac{4x+3}{6x-4}$, $x \neq \frac{2}{3}$, show that fof(x) = x for all $x \neq \frac{2}{3}$. Also, find the inverse of f.

**14.** If $\tan^{-1}x - \cot^{-1}x = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$, x > 0, find the value of x and hence find the value of $\sec^{-1}\left(\frac{2}{x}\right)$.

**15.** Using properties of determinants, prove that

$$\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = 4abc$$

**16.** If sin y = x sin (a + y), prove that

$$\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$$

**OR**

If $(\sin x)^y = x + y$, find $\frac{dy}{dx}$.

**17.** If $y = (\sec^{-1} x)^2$, x > 0, show that

$$x^2\left(x^2-1\right)\frac{d^2y}{dx^2} + \left(2x^3 - x\right)\frac{dy}{dx} - 2 = 0$$

**18.** Find the equation of the tangent and the normal to the curve $y = \frac{x - 7}{(x-2)(x-3)}$ at the point where it cuts the x-axis.

**19.** Find:

$$\int \frac{\sin 2x}{(\sin^2 x + 1)(\sin^2 x + 3)} dx$$

**20.** Prove that

$\int_a^b f(x)\,dx = \int_a^b f(a+b-x)dx$ and hence evaluate

$$\int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}}.$$

**21.** Solve the differential equation:

$$\frac{dy}{dx} = \frac{x+y}{x-y}$$

**OR**

Solve the differential equation:

$(1 + x^2)dy + 2xy\,dx = \cot x\,dx$

**22.** Let a, b and c be three vectors such that $|a| = 1$, $|b| = 2$ and $|c| = 3$.

If the projection of b along a is equal to the projection of c along a; and b, c are perpendicular to each other, then find $|3a - 2b + 2c|$.

**23.** Find the value of $\lambda$ for which the following lines are perpendicular to each other:

$$\frac{x-5}{5\lambda+2} = \frac{2-y}{5} = \frac{1-z}{-1};\quad \frac{x}{1} = \frac{y+\frac{1}{2}}{2\lambda} = \frac{z-1}{3}$$

Hence, find whether the lines intersect or not.

## SECTION D

Questions number 24 to 29 carry 6 marks each.

**24.** If $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 1 & -2 & 1 \end{bmatrix}$' find $A^{-1}$

Hence, solve the following system of equations:
$x + y + z = 6,$
$y + 3z = 11$
and $x - 2y + z = 0$

**OR**

Find the inverse of the following matrix, using elementary transformations:

$A = \begin{bmatrix} 2 & 3 & 1 \\ 2 & 4 & 1 \\ 3 & 7 & 2 \end{bmatrix}$

**25.** Show that the height of a cylinder, which is open at the top, having a given surface area and greatest volume, is equal to the radius of its base.

**26.** Find the area of the triangle whose vertices are (-1, 1), (0, 5) and (3, 2), using intergration.

**OR**

Find the area of the region bounded by the curves $(x - 1)^2 + y^2 = 1$ and $x^2 + y^2 = 1$, using integration.

**27.** Find the vector and cartesian equations of the plane passing through the points (2, 5, -3), (-2, -3, 5) and (5, 3, -3). Also, find the point of intersection of this plane with the line passing through points (3, 1, 5) and (-1, -3, -1).

**OR**

Find the equation of the plane passing through the intersection of the planes $r.\left( i + j + k\right) = 1$ and $r.\left( 2i + 3j - k\right) + 4 = 0$ and parallel to x-axis. Hence, find the distance of the plane from x-axis.

**28.** There are two boxes I and II. Box I contains 3 red and 6 black balls. Box II contains 5 red and 'n' black balls. One of the two boxes, box I and box II is selected at random and a ball is drawn at random. The ball drawn is found to be red. If the probability that this red ball comes out from box II is $\frac{3}{5}$, find the value of 'n'.

**29.** A company manufactures two types of novelty souvenirs made of plywood. Souvenirs of type A require 5 minutes each for cutting and 10 minutes each for assembling. Souvenirs of type B require 8 minutes each for cutting and 8 minutes each for assembling. There are 3 hours and 20 minutes available for cutting and 4 hours available for assembling. The profit is Rs.50 each for type A and Rs.60 each for type B souvenirs. How many souvenirs of each type should the company manufacture in order to maximize profit ? Formulate the above LPP and solve it graphically and also find the maximum profit.